Private Learning and Sanitization: Pure vs. Approx. Differential Privacy

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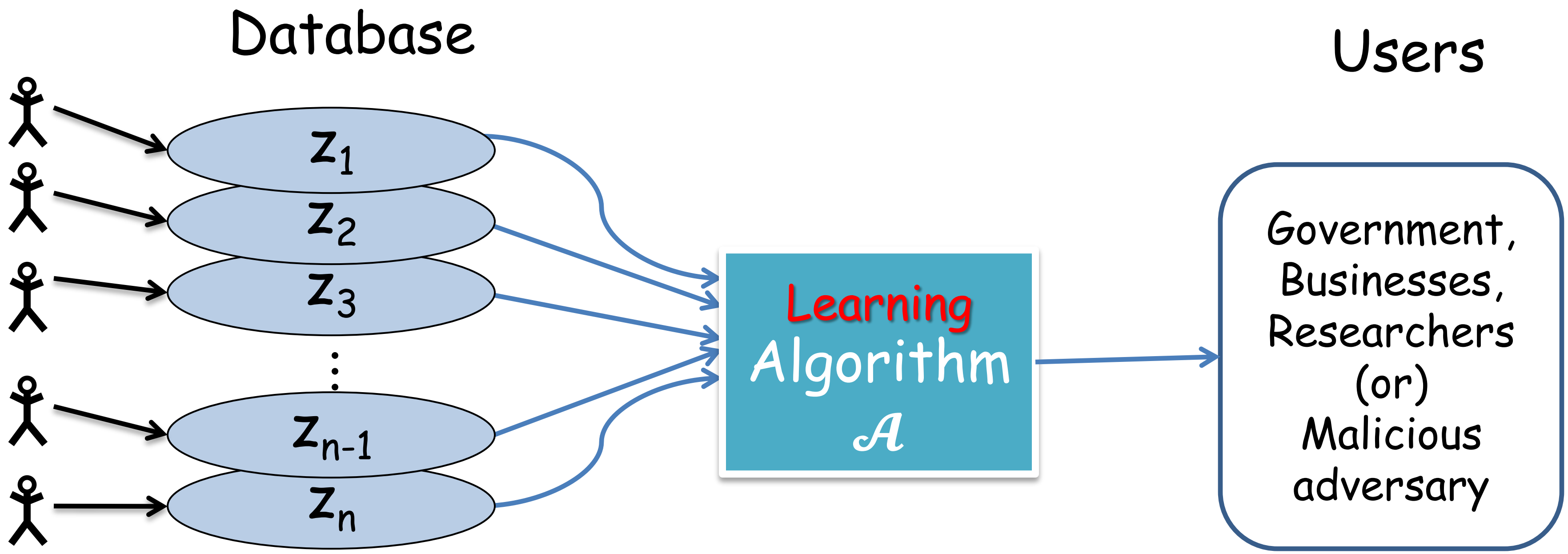
Join work with Amos Beimel and Kobbi Nissim

Why Private Learners?



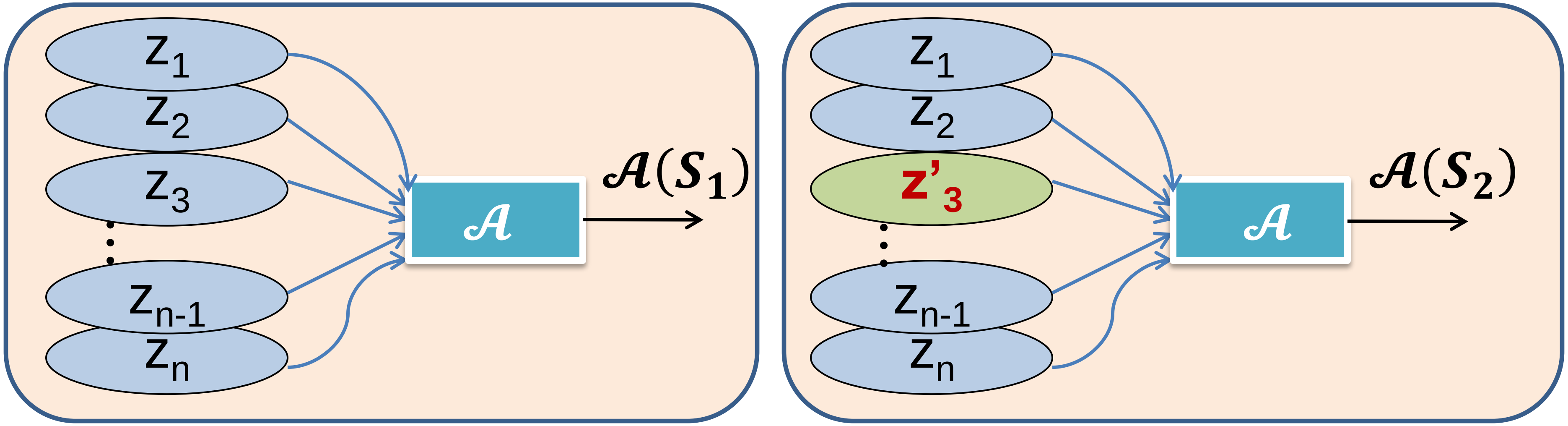
Often, this algorithmic task can be abstracted as a learning problem:

- Bank is interested in predicting (based on past customers) whether new customers are good/bad credit

Differential Privacy

Dwork, McSherry, Nissim, Smith 2006

Changing one record does not change the output distribution **"too much"**



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A (rand) algorithm $\mathcal{A}$ is differentially private if for all neighboring databases S_1, S_2 and for all sets of outputs F:

$$\Pr[\mathcal{A}(S_1) \in F] \approx \Pr[\mathcal{A}(S_2) \in F]$$

Pure Differential Privacy

Dwork, McSherry, Nissim, Smith 2006

Changing one record does not change the output distribution **"too much"**

A (rand) algorithm $\mathcal{A}$ is ϵ differentially private if for all neighboring databases S_1, S_2 and for all sets of outputs F:

$$\Pr[\mathcal{A}(S_1) \in F] \leq e^{\epsilon} \cdot \Pr[\mathcal{A}(S_2) \in F]$$

Approx. Differential Privacy

Dwork, McSherry, Nissim, Smith 2006

Dwork, Kenthapadi, McSherry, Mironov, Naor 2006

Changing one record does not change the output distribution **"too much"**

A (rand) algorithm $\mathcal{A}$ is $(\epsilon, \boldsymbol{\delta})$ differentially private if for all neighboring databases S_1, S_2 and for all sets of outputs F:

$$\Pr[\mathcal{A}(S_1) \in F] \leq e^{\epsilon} \cdot \Pr[\mathcal{A}(S_2) \in F] + \boldsymbol{\delta}$$

Our Results:

- Sample complexity of Private Learning and Sanitization can be **drastically smaller** if we settle for **approximate** differential privacy.

- **Label Privacy** [Chaudhuri and Hsu 2011]
  Learning model with weakened privacy demands.
  We settle the question of sample complexity: **O(VC)**.
  - **Same as non-private learning.**
  - **Not is this talk.**

- Natural connection between Private Learning and Sanitization, leads to lower bounds on Sanitization.
  - **Not in this talk.**

What is Private Learning?

Kasiviswanathan, Lee, Nissim, Raskhodnikova, Smith 08

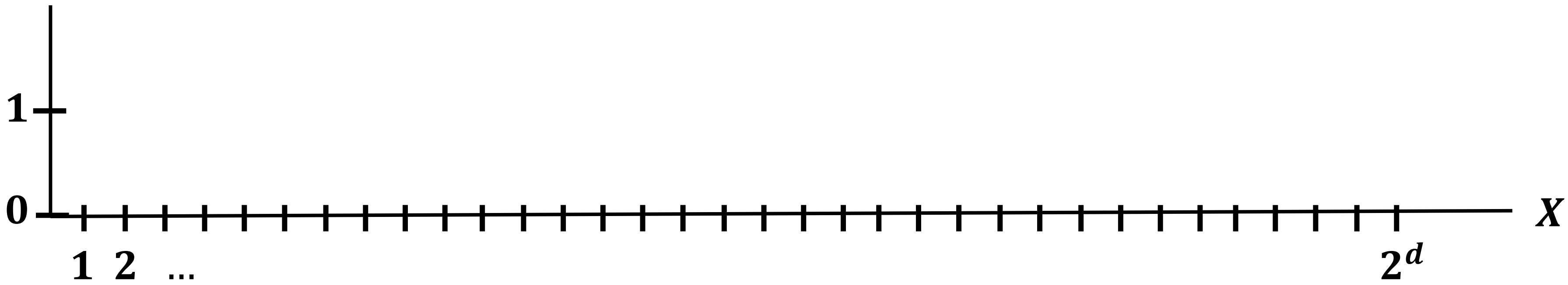
Definition:

PAC Learning
\+ Differential Privacy

Private Learning

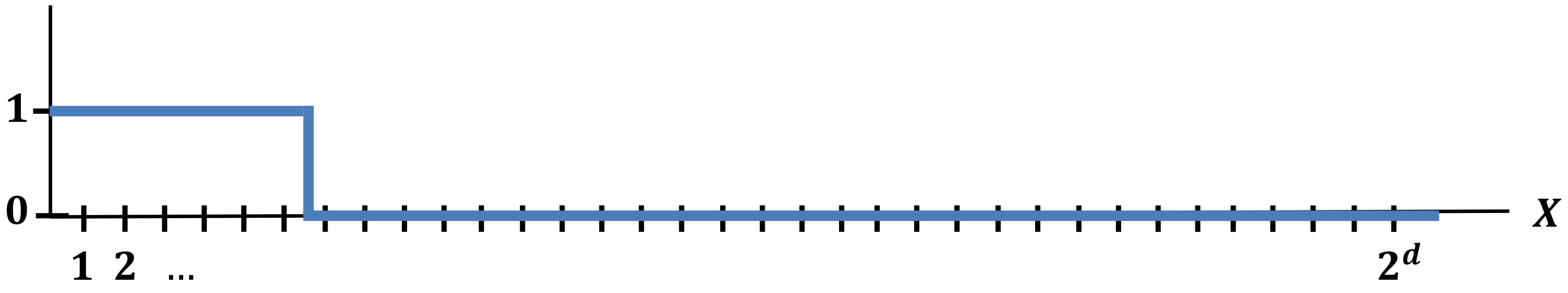
"PAC" Model [Valiant 84]

- Domain X.



“PAC” Model [Valiant 84]

- Domain X.
- Set $\mathcal{C}$ of boolean functions over X.
    - for example: $\mathbf{INTERVAL}_d$



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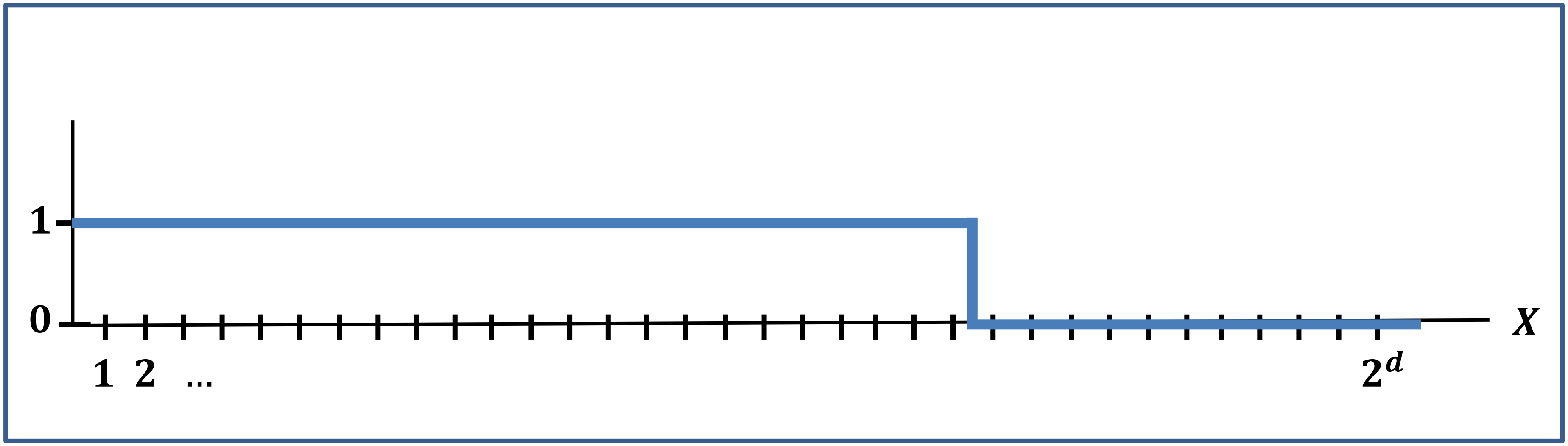

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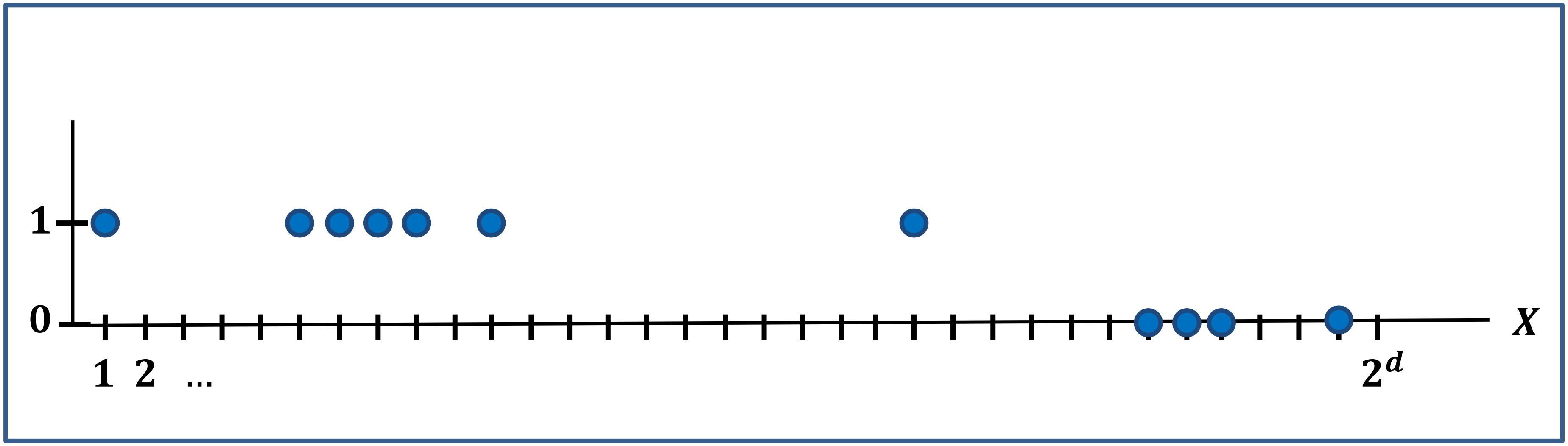

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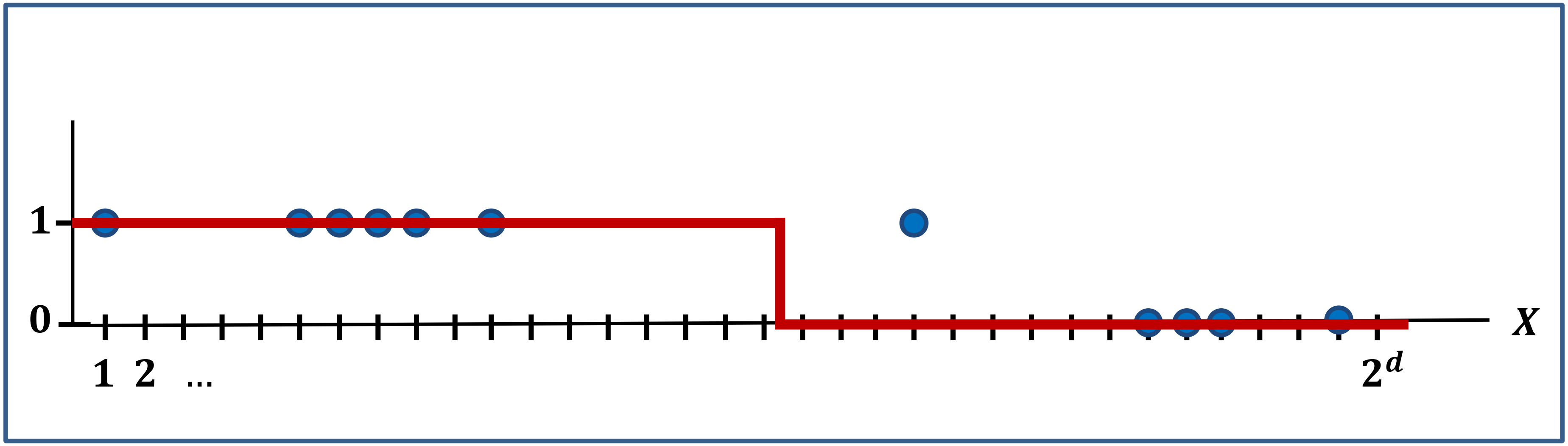

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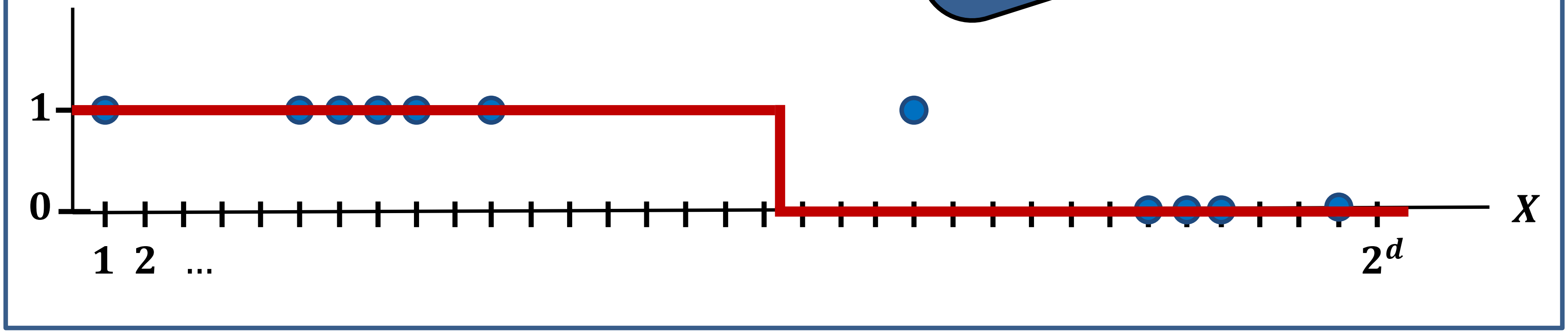
PROPER if $h \in \mathcal{C}$



Related work in Private Learning (partial list)

[BDMN 05] First private learning algorithms. SQ based.

[KLNRS 08] Define private learning, and showed:
Every class $\mathcal{C}$ can be privately learned using $\log|\mathcal{C}|$ labeled samples.

[BKN 10] Sample complexity of private learning.

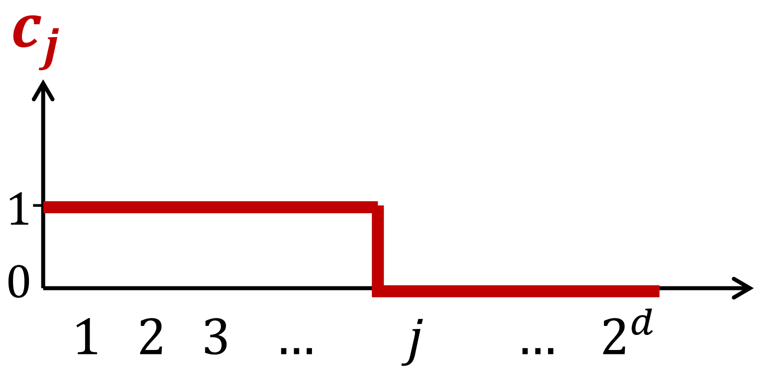
[CH 11] Learning in continuous domain, label privacy.

[CM 08, CMS 11, KST 12] Machine learning.

[BLR 08, DNRRV 09, …] Synthetic Data.

[DRV 10] Private Boosting.

Running Example: $\mathrm{INTERVAL_d}$



$c_j(x) = 1 \iff x < j$

Facts:

- non-private **proper** learner with $O(1)$ samples.
- ϵ-private **proper** learner: $\Theta(d)$ samples [BBKN 10].

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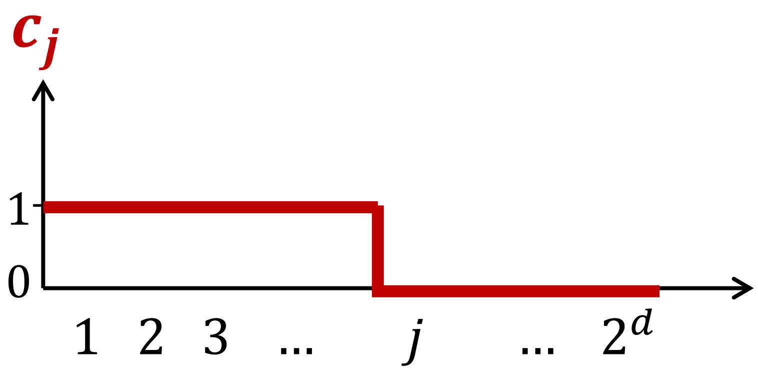

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We show:

(ϵ, δ)-private **proper** learner with $2^{O(\log^* d)}$ samples.

Privately Learning intervals: Ideas and Intuition.

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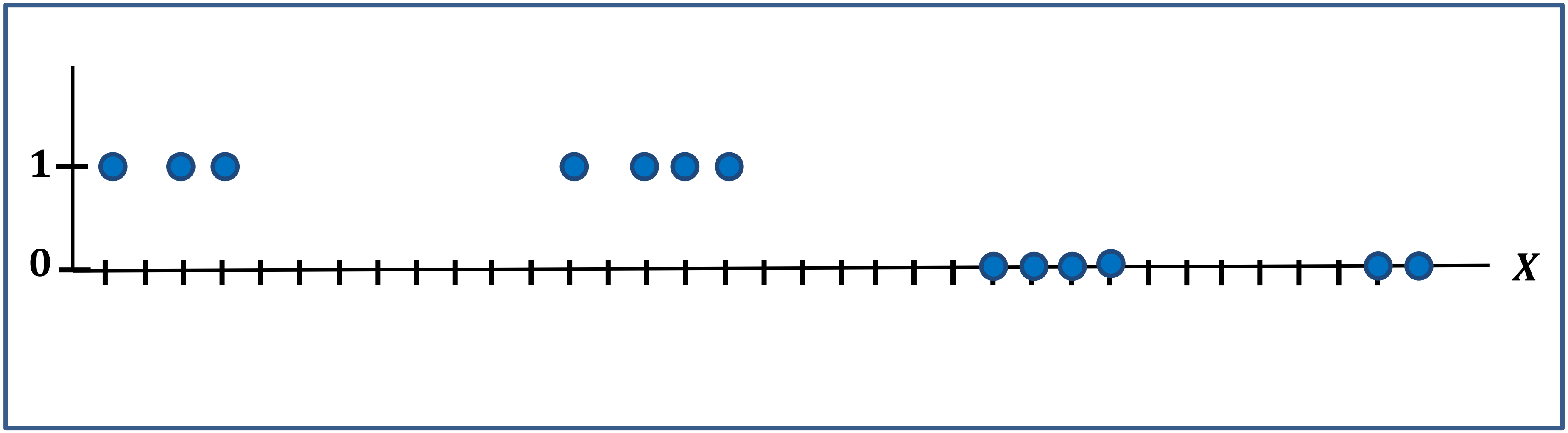
(ϵ, δ)-private **proper** learner with $2^{O(\log^* d)}$ samples.

The Goal:

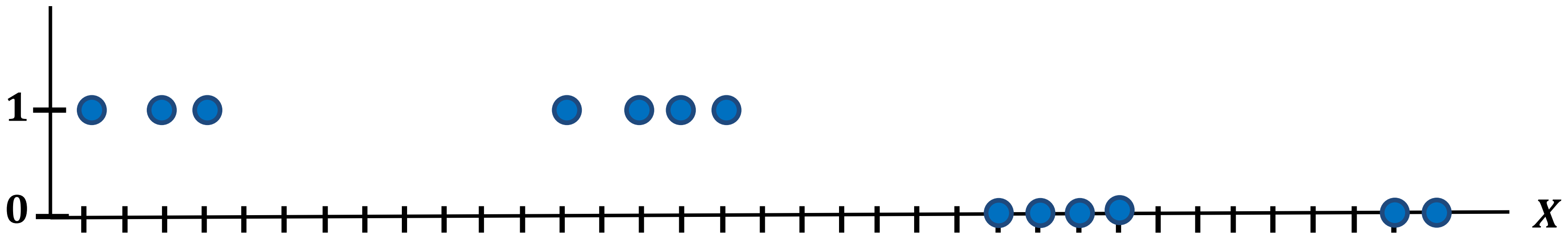
Given a labeled sample, choose a concept with small error.



4-good interval G

Assume we can (privately) obtain an interval $G \subseteq X$ s.t.

- Contains "a lot" of ones, **and** "a lot" of zeroes.
- Every interval $I \subseteq X$ of **length** $\leq |G|/4$ either does **not contain** "too many" ones **or** does **not contain** "too many" zeroes.



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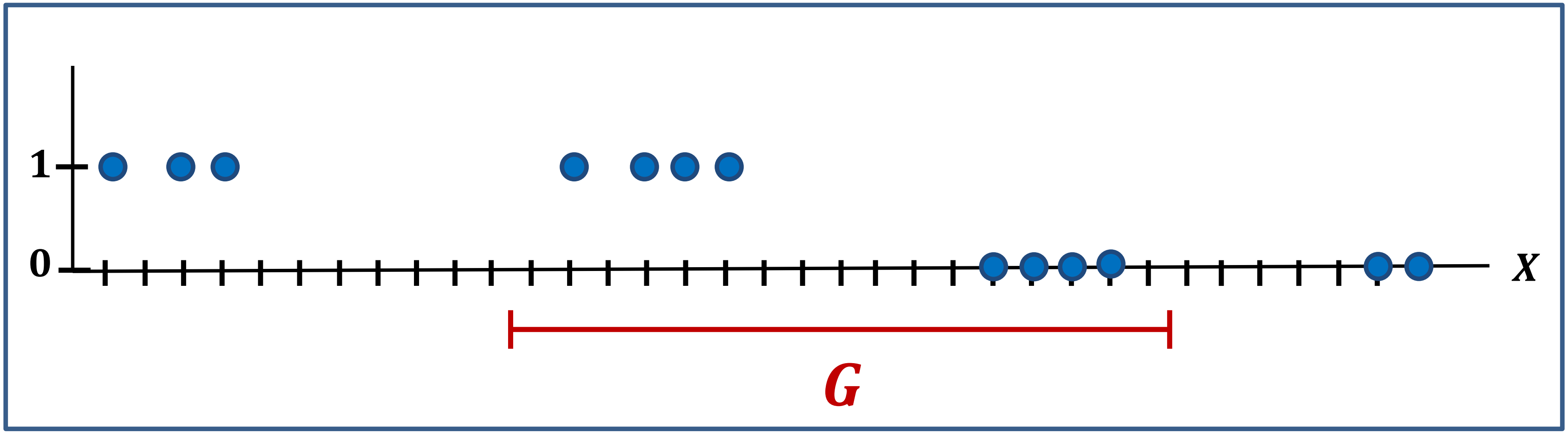

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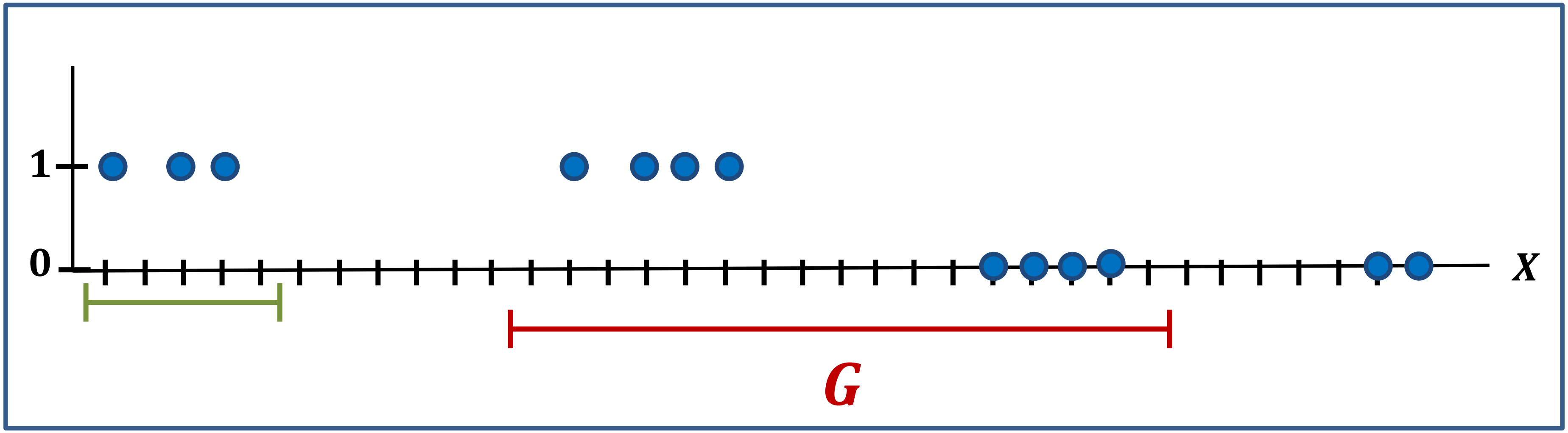

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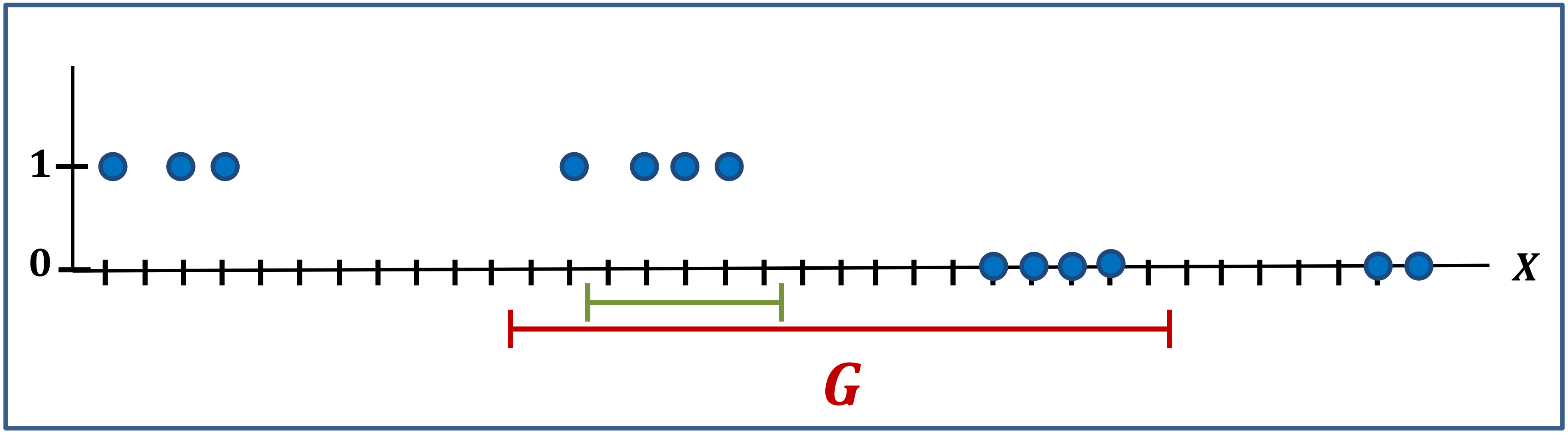

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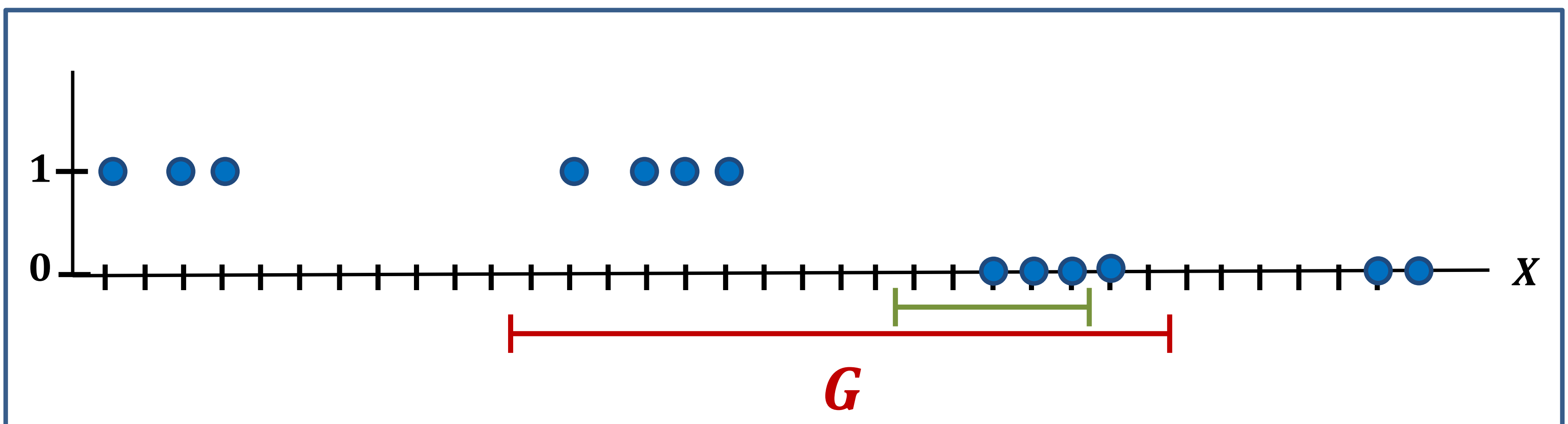

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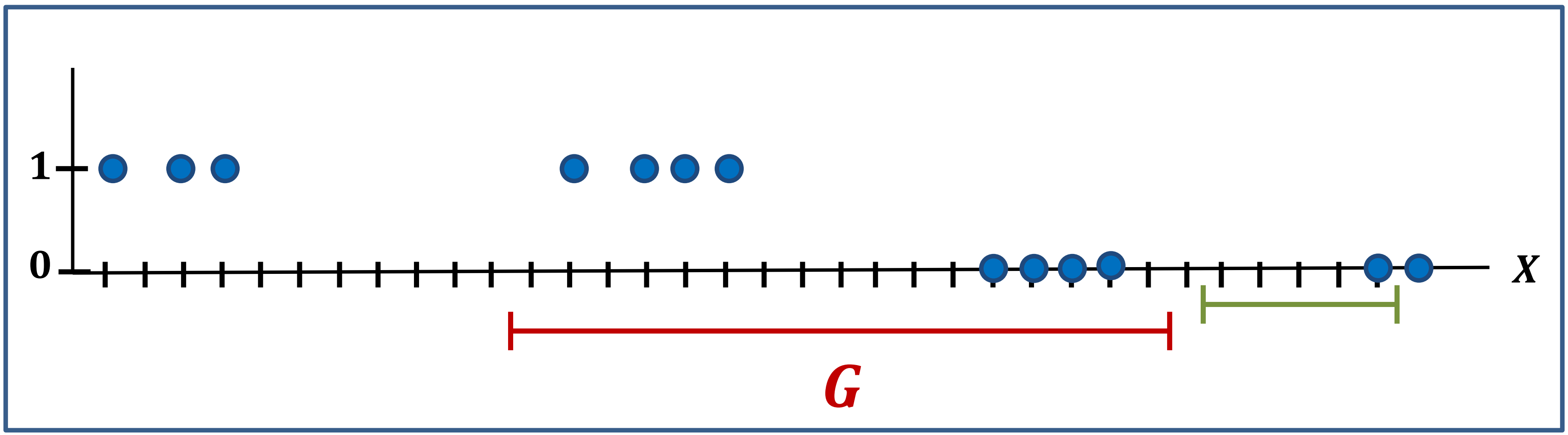

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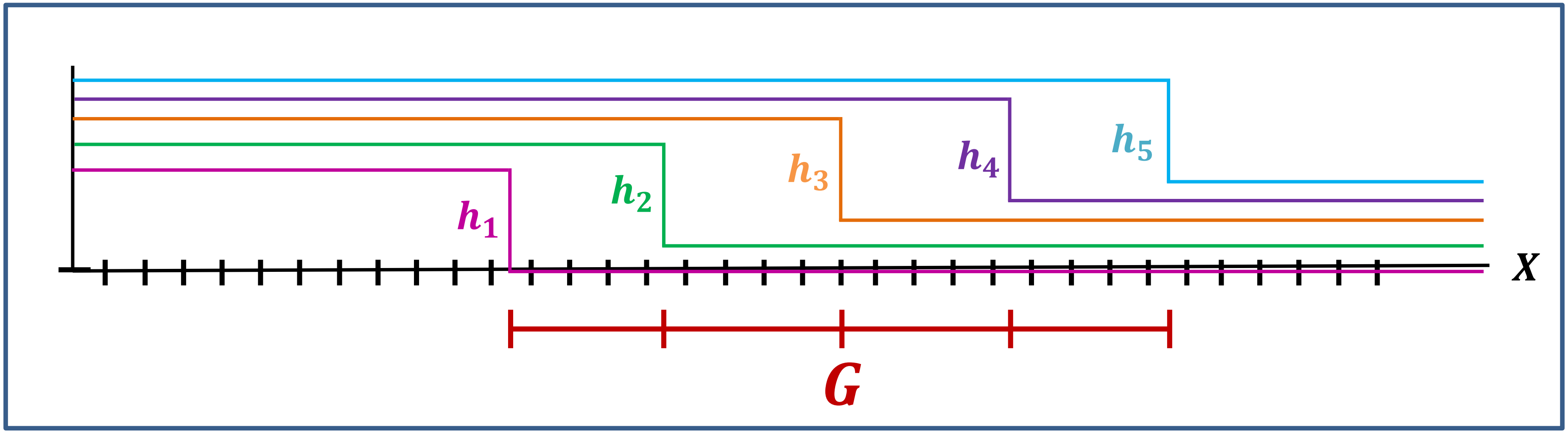
- Divide G into **4** equal intervals, and define **5** "equally spread" concepts in G.
- At least one concept has small error.



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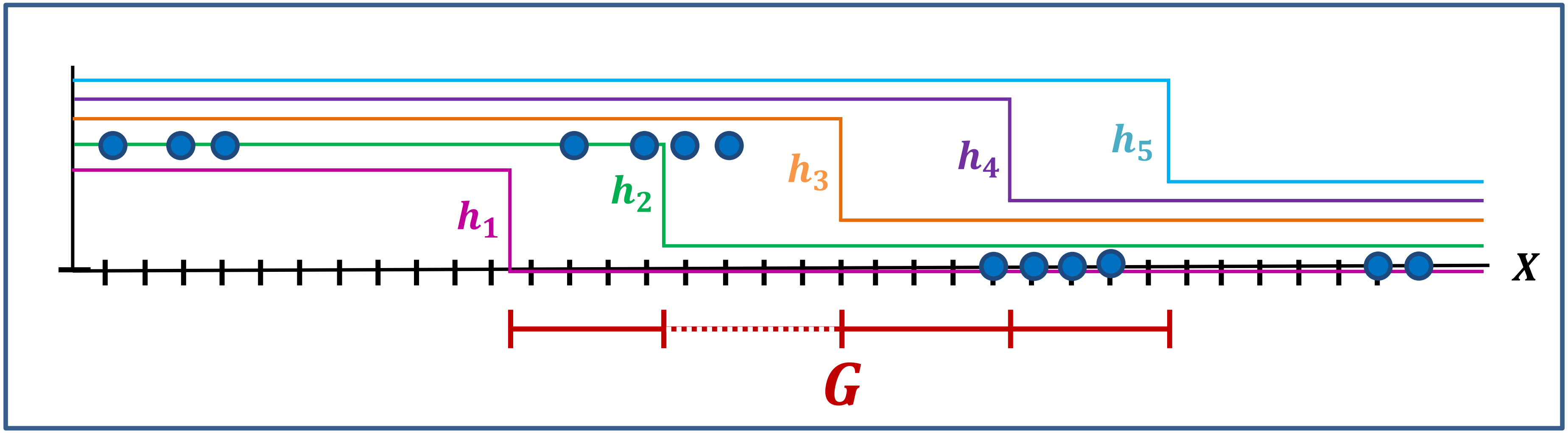

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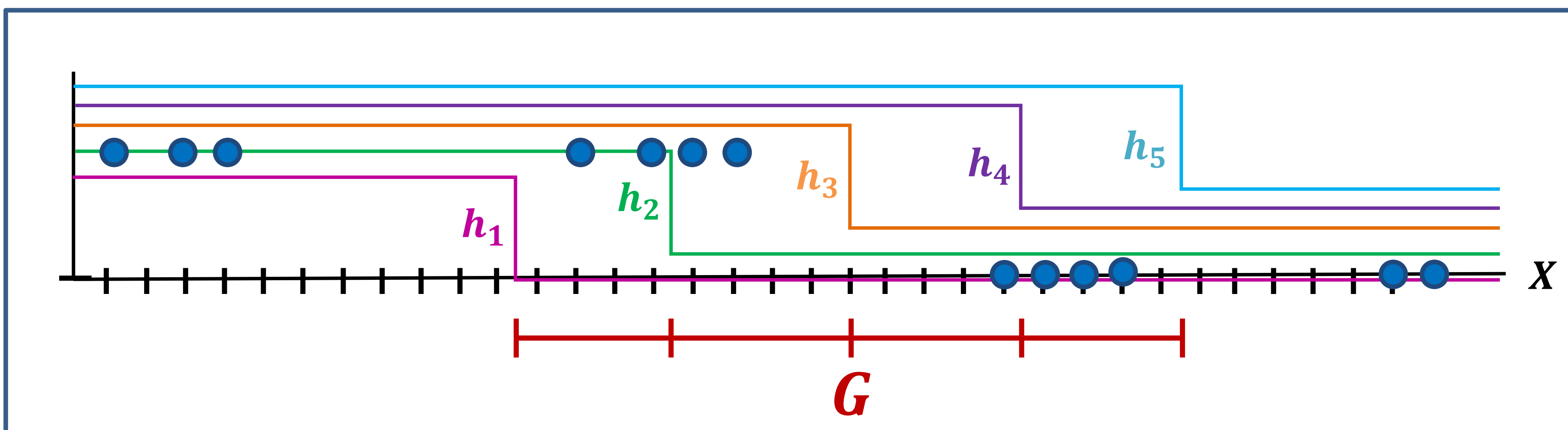

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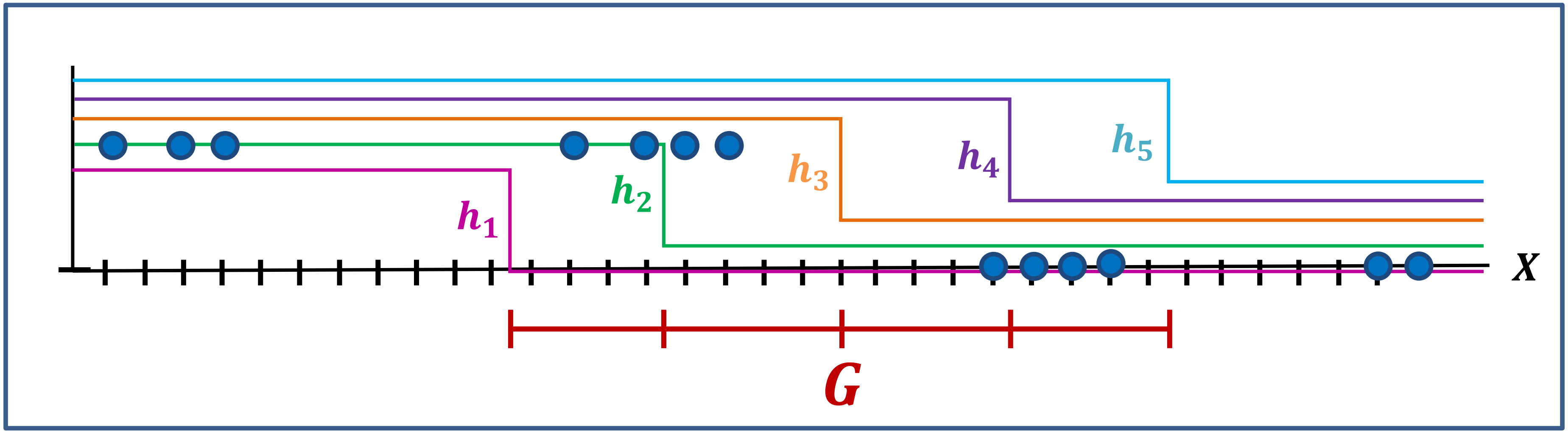

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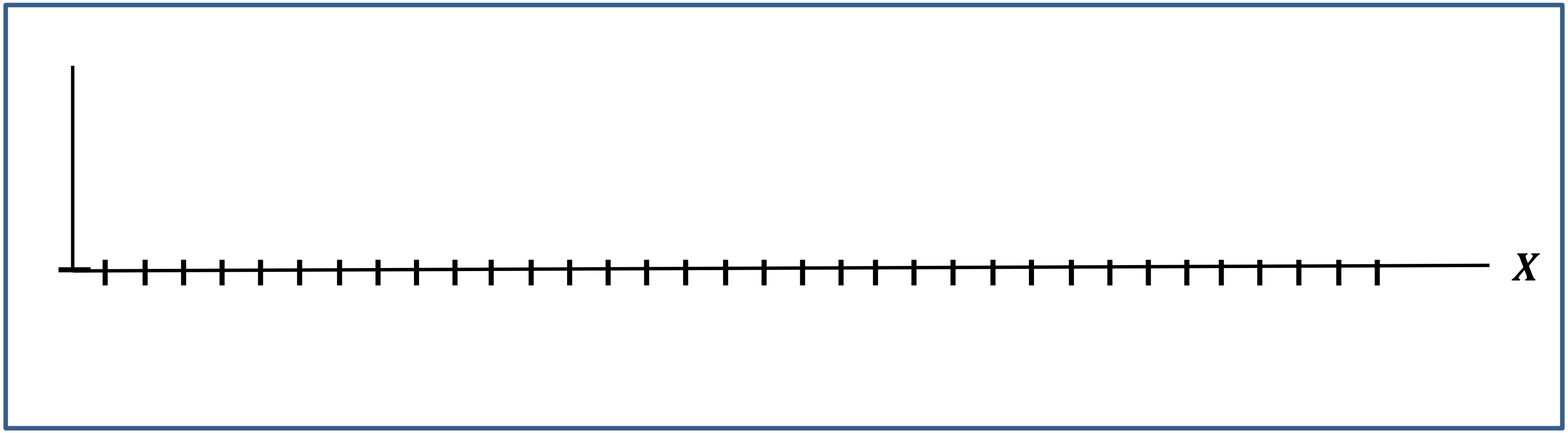

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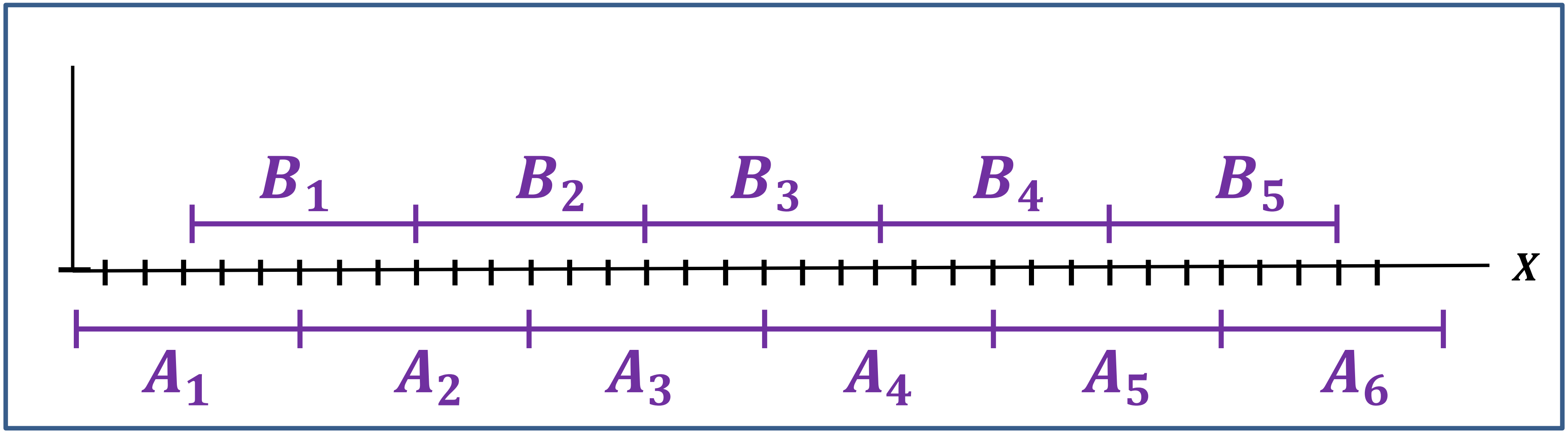

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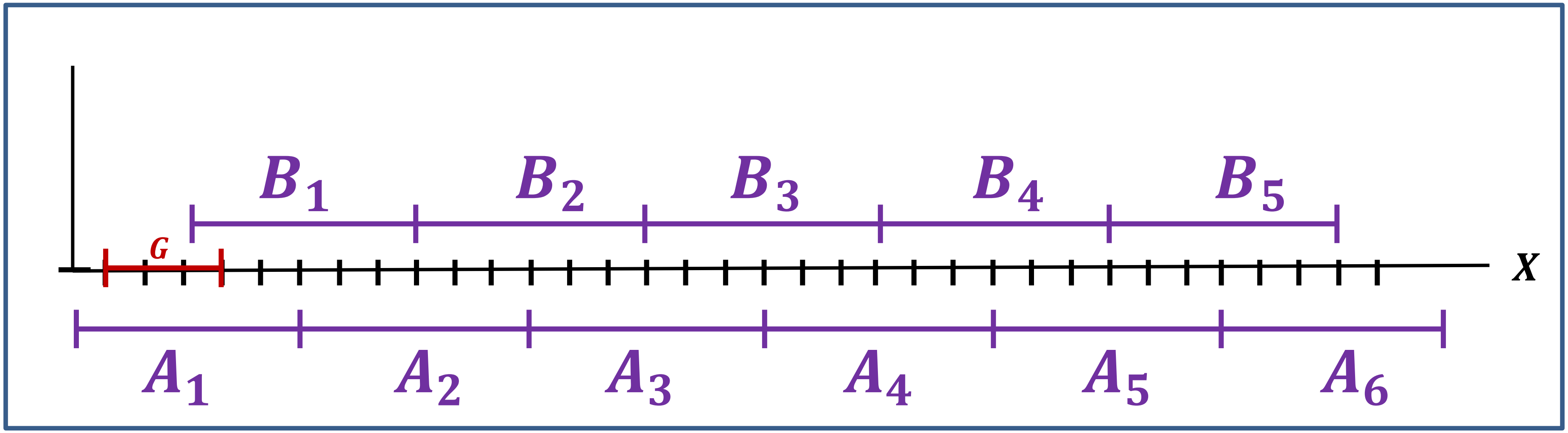

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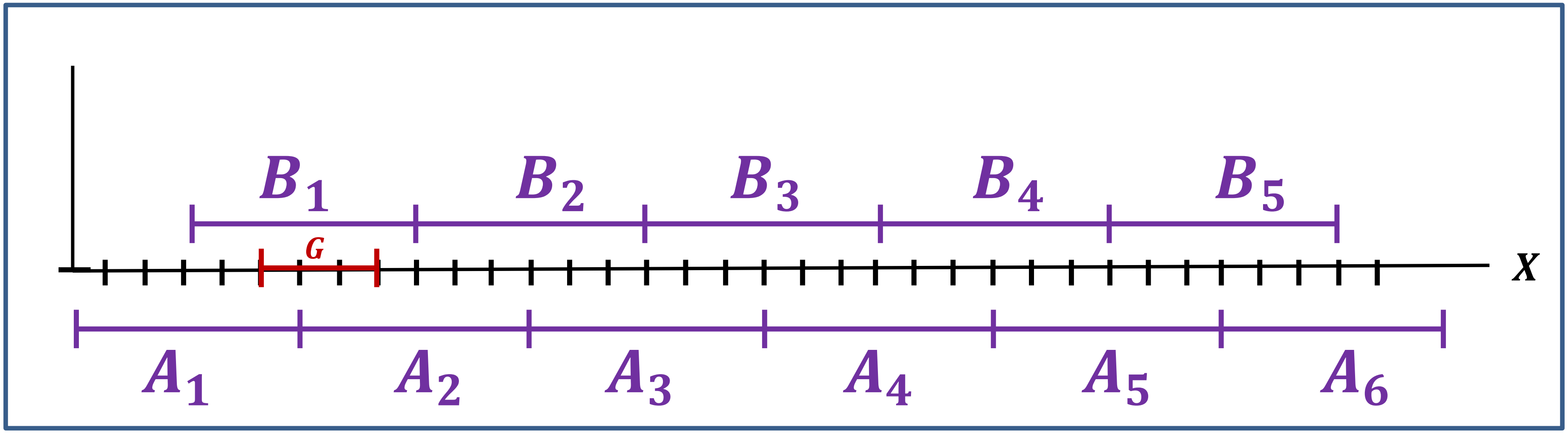

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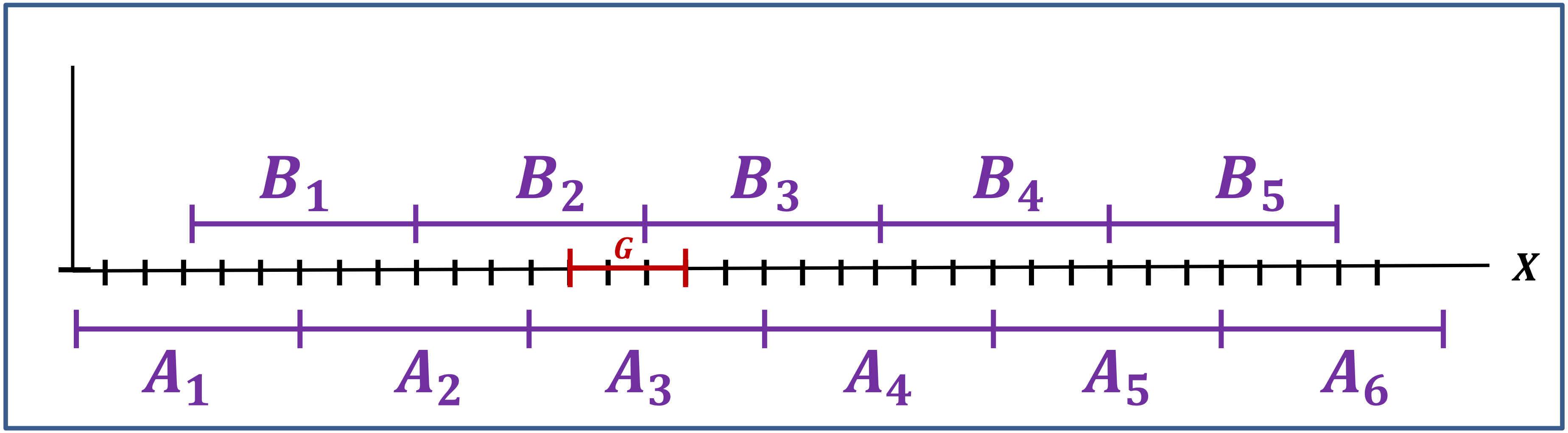

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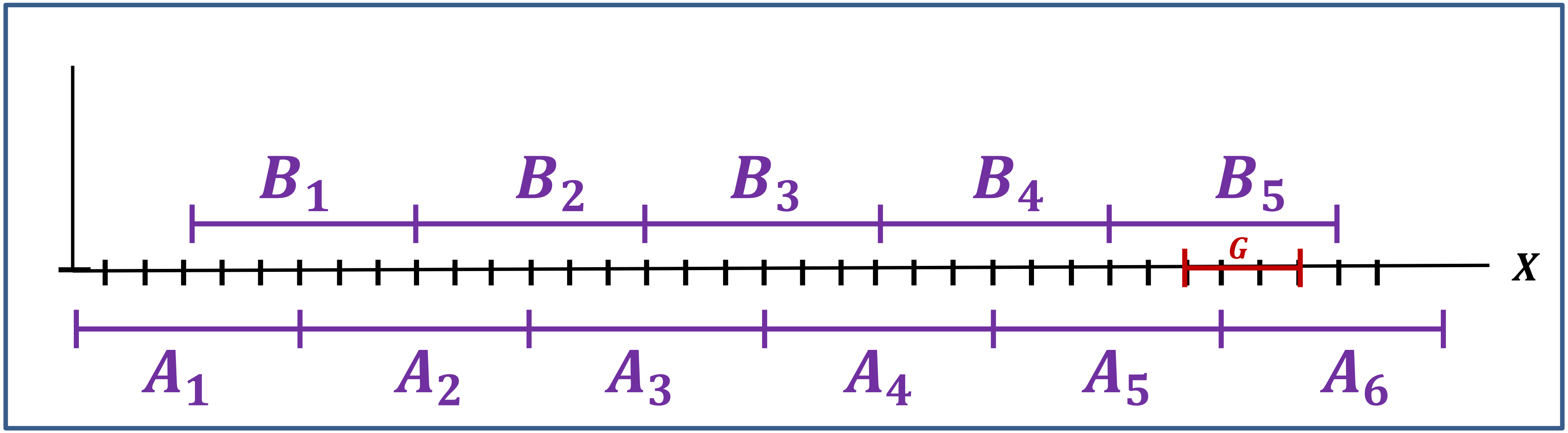

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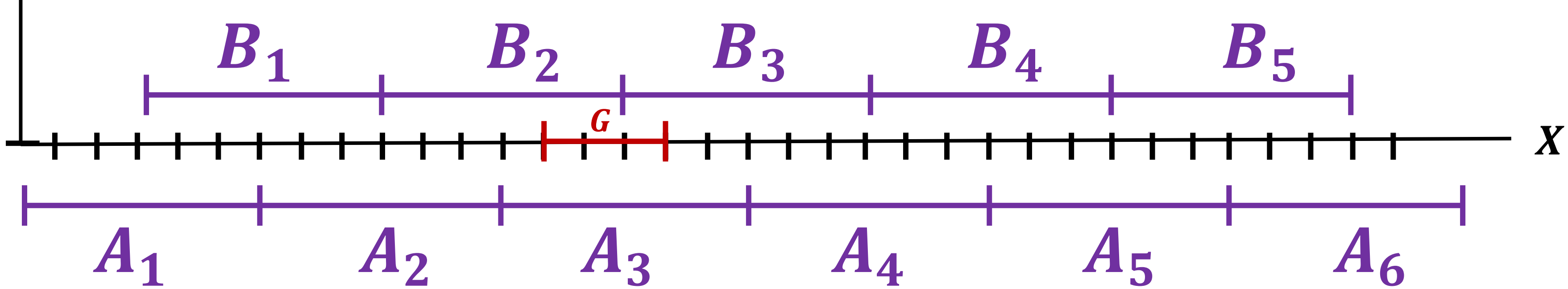
- Say $G \in A_3$. Then A_3 contains "lots" of ones <u>and</u> zeroes.
- Every other A_i cannot contain both ones and zeroes.
- Look for A_i with "lots" of ones <u>and</u> zeroes.



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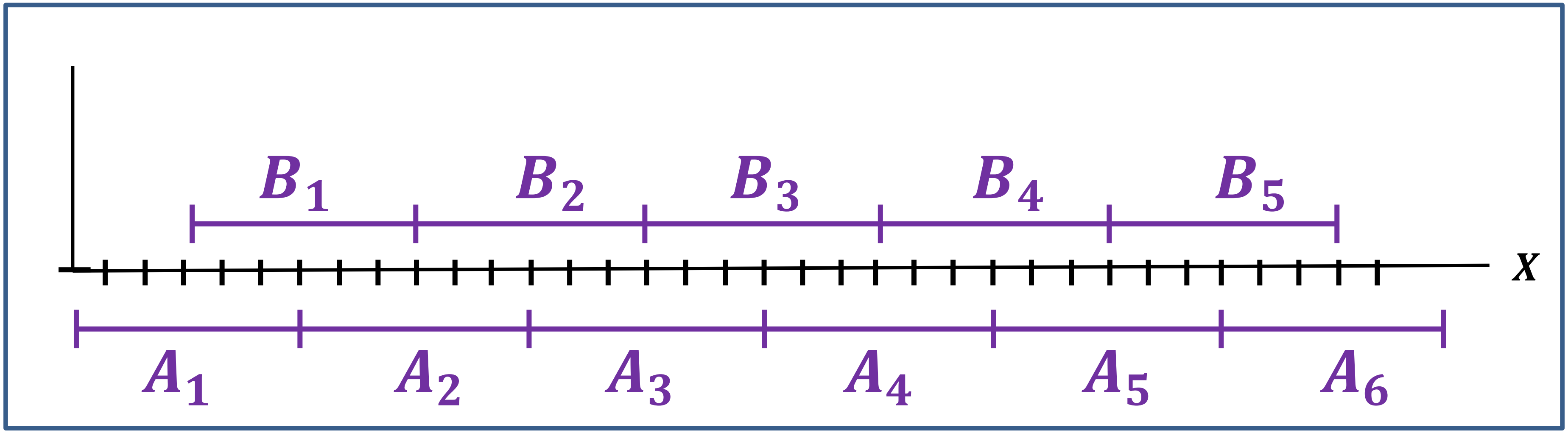

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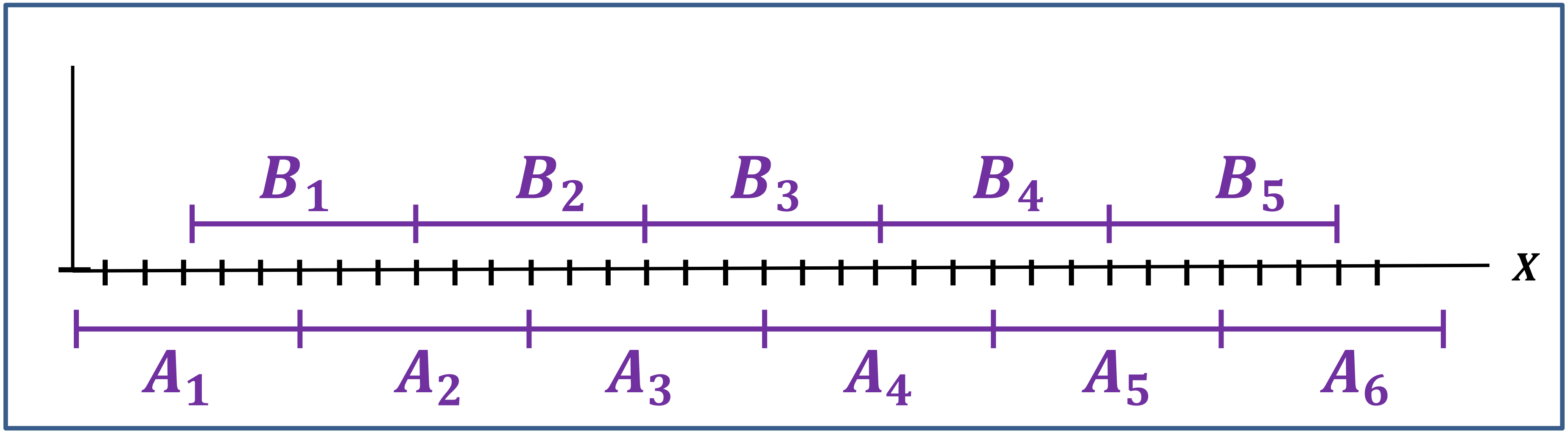

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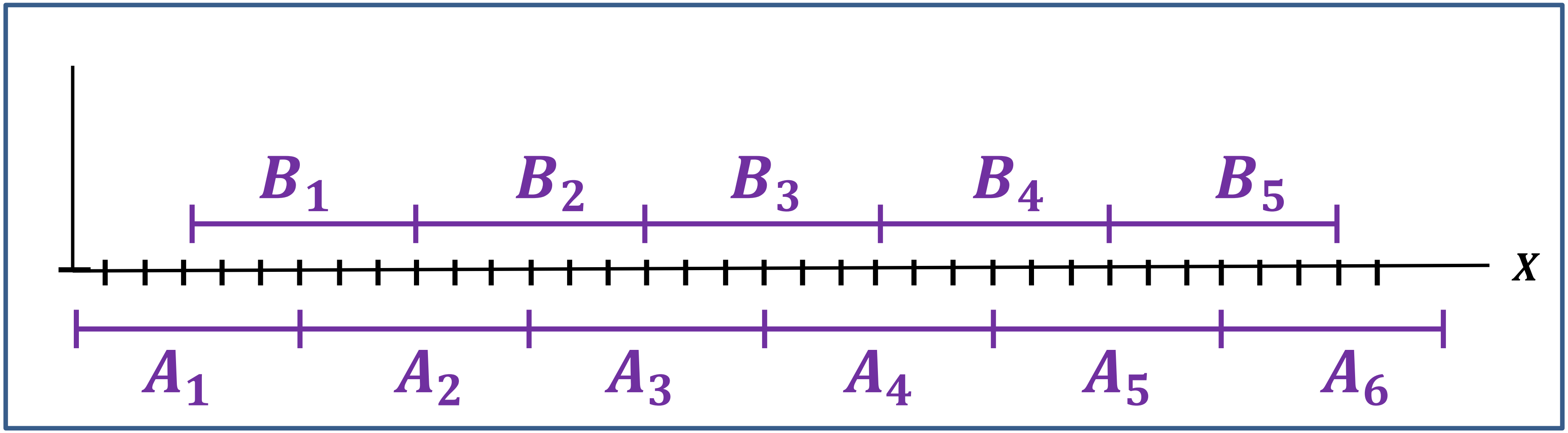
- The chosen interval is of length $2|G|$ $\Longrightarrow$ **4-good !**



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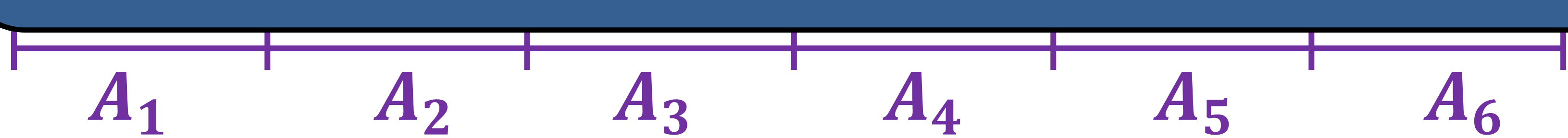
Conclusion: suffices to find a length J of a 2-good interval.



Computing the length J

Easy solution:

- Noisy binary search on $\mathbf{0} \leq \mathbf{J} \leq \mathbf{2^d}$.
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Better solution:

- Noisy binary search on the power $\mathbf{0} \le j \le \log d$ of a 2-good interval of length J= $\mathbf{2}^{j}$.
- $\log d$ noisy comparisons requires $\log d$ samples.

Computing the length J

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- $\mathbf{log}\, \boldsymbol{d}$ noisy comparisons requires $\mathbf{log}\, \boldsymbol{d}$ samples.

In the paper:

Use recursion on binary search and significantly reduce the costs.

Theorem:

There exists an (ϵ, δ)-private learner for $\mathbf{INTERVAL}_{\boldsymbol{d}}$ with sample complexity $\mathbf{2}^{\mathbf{log}^* \boldsymbol{d}}$.

Summary and Open Problems

- **What we see:**

  Efficient (ϵ, δ)-private learner for $\mathbf{INTERVAL}_d$ with low sample complexity.

  - This **separates** the sample complexity of (ϵ, δ)-private and ϵ-private learners.

- **Other results:**
  - Efficient $(\epsilon, \boldsymbol{\delta})$-private for other concept classes with **even lower** sample complexity (**independent of the domain**).
  - Similar results for Data Sanitization.

- **Open problem:**

  Lower bounds on the sample complexity of $(\epsilon, \boldsymbol{\delta})$-private learners?